International Journal of Mathematics and Computer Applications Research (IJMCAR)
ISSN(P 2249-6955; ISSN(E): 2249-8060
Vol. 5, Issue 3, Jun 2015, 65-74
TJPRC Pvt. Ltd.

A THEORTICAL STUDY OF BOUNDARY LAYER FLOW OF WATER BASED NANOFLUIDS OVER A FLAT PLATE PLACED IN A MAGNETIC FIELD

PREETI AGARWALA & R KHARE

Department of Mathematics & Statistics, Sam Higginbottom Institute of Agriculture, Technology & Sciences
Allahabad, Uttar Pradesh, India

ABSTRACT

The objective of the present paper is to study the boundary layer flow of different water based nanofluids over a flat plate placed in a magnetic field. A similarity analysis is performed to reduce the governing equations of continuity and momentum into nonlinear differential equations which are subsequently solved numerically using fourth order Runge—Kutta method with shooting techniques.. The numerical value for thermal conductivity, kinematic viscosity, heat capacity, density, and thermal diffusivity for different type of nanofluids with respect to different values of volume fraction are evaluated. Along with this the results for the dimensionless velocity, temperature, nanofluid solid volume fraction and skin friction coefficient in the presence of magnetic parameter have been investigated. The results are displayed graphically to show the interesting aspects of nanofluids.

KEYWORDS: Nanofluids, MHD Boundary Layer, Flat Plate, Similarity Solution

INTRODUCTION

Flow of an incompressible viscous fluid over moving surfaces has an important bearing on several technical applications such as in metallurgy and chemical processes industries as they involve heat transfer by means of a flowing fluid in either laminar or turbulent regimes..Nanofluids are fluids that contain small volumetric quantities of nanometer—sized particles, called nanoparticles. The nanoparticles used in nanofluids are typically made of metals(Al, Cu, Ag, Au, Fe), oxides (Al_20_3, CuO, $Ti0_2$), metal carbides(SiC), non-metals(graphite carbon nanotubes), and others. Common base fluids include water, ethylene glycol and oil. Nanofluids commonly contain up to a 5%volume fraction of nanoparticles to see effective heat transfer enhancements. Thermophysical properties of nanofluids have been enormously studied, to mention some, Rudyak et al. [1] studied the thermal conductivity of nanofluids. Sakiadis [2] studied the problem of forced convection along an isothermal moving plate. Ahmad and Pop [3] discussed the problem of mixed convection boundary layer flow from a vertical flat plate embedded in a porous medium filled with nanofluids. Magnetohydrodynamics (MHD) is the study of the flow of electrically conducting fluids in a magnetic field. Recently, the application of magnetohydrodynamics in the polymer industry and metallurgy has attracted the attention of many researchers. Several researchers [10, 11, 15] investigated the MHD flow of nanofluids. Makinde and Aziz [4] studied MHD flow over an inclined radiating plate with the temperature-dependent thermal conductivity, variable reactive index, and heat generation. Hamad et al, [8] investigated the magnetic field effects on free convection flow of a nanofluid past a vertical semi-infinite flat plate. Khan and Pop [11] presented a similarity solution for the free convection boundary layer flow past a horizontal flat plate embedded in a porous medium filled with a nanofluid. Devi and Andrew [6, 7] studied Laminar boundary layer flow of nanofluids over a flat plate. Khare and Srivastava [12,13,14] investigated MHD flow of a visco elastic dusty fluid

through a porous medium induced by the motion of a semi infinite Plate.

The aim of the present paper is to study the combined effect of uniform transverse magnetic field on the flow of different nanofluids over a flat plate. The governing highly nonlinear partial differential equation of momentum and energy fields have been simplified by using a suitable similarity transformations and then solved numerically using fourth order Runge—Kutta method with shooting techniques. The effects of the governing parameters on the velocity and temperature have been discussed and presented in tables and graphs.

MATHEMATICAL FORMULATION

We consider a steady two-dimensional laminar boundary layer flow of nanofluids over a flat plate placed in a magnetic field. The continuity, momentum and energy equations are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}-\sigma\frac{B_0^2(x)}{\rho_{nf}}u \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Where, x and y are the coordinates along and perpendicular to the plate, u and v are the velocity components in the x and y directions respectively.

$T =$ Temperature of the nanofluid

$T_\infty =$ Temperature of the nanofluid far away from the plate

$B_0 =$ The uniform magnetic field strength

σ = Electrical conductivity of base fluid.

μ_{nf} = Dynamic viscosity of the nanofluid

$\rho_{nf} =$ Effective density of the nanofluid

$\alpha_{nf} =$ Thermal diffusivity of the nanofluid

The effective density of the nanofluid is given by

$$\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s \tag{4}$$

And the thermal diffusivity of the nanofluid is

$$\alpha_{nf} = \frac{K_{nf}}{(\rho C_p)_{nf}} \tag{5}$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}} \tag{6}$$

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi\,(\rho C_p)_s \tag{7}$$

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \tag{8}$$

Here $\upsilon_f, \mu_f, \rho_f, k_f$ are the Kinematic viscosity, dynamic viscosity, density and thermal conductivity of the base fluid respectively; $\rho_s, k_s, (\rho C_p)_s$ are the density thermal conductivity and heat capacitance of the nanoparticles respectively; ϕ is the solid volume fraction of nanoparticles and k_{nf} is thermal conductivity of the nanofluid.

The appropriate initial and boundary conditions for the above problem are given by

$$y = 0;\ u = v = 0\ , T = T_w$$

$$y = \infty;\ \ u = U_\infty\ , T = T_\infty \tag{9}$$

Two different types of nanofluids are considered for study namely, Copper oxide (CuO), and Titania (TiO2), with water as the base fluids (i.e. with a constant Prandtl number Pr = 6.58). The thermophysical properties of the nanofluids were assumed to be functions of the volume fraction are calculated using the different equations given above.

The thermo physical properties of base fluid and nano particles are given in Table 1

Table 1: Thermophysical Properties of Base Fluid and Nano Particles

Physical Properties	Water (H2O)	CopperOxide (CuO)	Titania (TiO2)
$\rho(kg/m^3)$	997.1	63.20	4250
$C_\rho(J/kg\ \kappa)$	4179	531.8	686.2
$K(w/m\kappa)$	0.163	76.5	8.954

SOLUTION OF THE PROBLEM

To simplify the mathematical analysis of our study, introduce the following similarity transformations

$$\psi(x,y) = f(\eta)(U_\infty v_{nf}\, x)^{1/2} \qquad\qquad u = U_\infty f'(\eta)$$

$$v=\frac{1}{2}\left(\frac{\nu_{nf}U_\infty}{x}\right)^{1/2}\left(\eta\, f'(\eta)-f(\eta)\right)\quad \eta=y(U_\infty/\nu_f x)^{1/2}$$

$$\theta(\eta)=\frac{T-T_\infty}{T-T_w} \tag{10}$$

Where $\psi(x,y)$ is the stream function with? $u=\frac{\partial\psi}{\partial y}, v=-\frac{\partial\psi}{\partial x}$

$\theta(\eta)$ = dimensionless temperature.

Making use of equation (10) in equation (2) and (9) lead to the following non –dimensional non –linear differential equation

$$f''' + ff'' - Mf' = 0 \tag{11}$$

Similarly using equation (10) in equation (3) under boundary conditions (9)

The energy equation reduces to

$$\theta'' + \Pr\left\{E_c\left(f''^2 + Mf'^2\right) + f\theta'\right\} = 0 \tag{12}$$

And the corresponding boundary conditions (9) become

$$\eta=0,\ y=0,\ f=0,\ f'=0,\ \theta=1$$

$$\eta\to\infty,\ y\to\infty,\ f'=1,\ \theta=0 \tag{13}$$

$M=\frac{2\sigma x\beta_0^2}{\rho u}$ The magnetic parameter

$\Pr=\frac{\nu}{\alpha}$ is the Prandtl number

$E_c=\frac{U^2}{c_\rho(T_\omega-T_\infty)}$ Is the Eckert number

$(\Pr)_{nf}=\frac{\mu_{nf}(C_\rho)_{nf}}{\kappa_{nf}}$ is the Prandtl number of the nanofluid

The physical quantities of interest in this problem are the local skin friction coefficient C_f and the Nusselt number Nu, which are defined as

$$C_f = \frac{\tau_w}{\rho_f U^2} \qquad Nu = \frac{q_w}{k_f (T_w - T_\infty)} \tag{14}$$

Where,

τ_w is the surface shear stress and q_w is the surface heat flux,which are given by

$$\tau_w = \mu_{nf} \left(\frac{\partial u}{\partial y} \right)_{y=0} \qquad q_w = -k_{nf} \left(\frac{\partial T}{\partial y} \right)_{y=0} \tag{15}$$

Using the similarity variable (9),we obtain in non dimensional form as

$$\mathrm{Re}_x^{1/2} C_f = \frac{1}{(1-\phi)^{2.5}} f''(0) \qquad \mathrm{Re}_x^{-1/2} Nu = -\frac{k_{nf}}{k_f} \theta'(0) \tag{16}$$

$\mathrm{Re}_x = u_w x / v_f$ Is the local Reynolds number

NUMERICAL SOLUTION

The non-Linear boundary value problem represented by equations (11)-(12) is solved numerically subject to boundary conditions using the fourth order Runge-Kutta method with shooting technique. The numerical solutions are obtained for several values of the governing parameters i.e., nanoparticle solid volume fraction parameter and magnetic field parameter, dimensionless velocity and dimensionless temperature. The numerical values of the density, viscosity, heat capacity, Thermal Conductivity kinematic viscosity for copper oxide – water, titania – water, nanofluids for different values of volume fraction are also evaluated as presented in Table (2) and Table (3).

Table 2: Thermophysical Properties of Copper Oxide-Water Nanofluid

Φ	ρ_{nf}	$(C_\rho)_{nf}$	$(\mu)_{nf}$	$(k)_{nf}$	$(\mathrm{Pr})_{nf}$
0	997.1	4179	0.001002	0.613	6.830927
0.01	1050.329	4142.528	0.001027	0.631129	6.744144
0.02	1103.558	4106.056	0.001054	0.64962	6.661443
0.03	1156.787	4069.584	0.001081	0.668482	6.582621
0.04	1210.016	4033.112	0.00111	0.687727	6.507493
0.05	1263.245	3996.64	0.001139	0.707368	6.435889
0.06	1316.474	3960.168	0.00117	0.727415	6.36765
0.07	1369.703	3923.696	0.001201	0.747883	6.302631
0.08	1422.932	3887.224	0.001234	0.768785	6.240696
0.09	1476.161	3850.752	0.001268	0.790133	6.181721
0.1	1529.39	3814.28	0.001304	0.811944	6.12559

Table 3: Thermo Physical Properties of Titania _Water nonofluid

Φ	ρ_{nf}	$(C_\rho)_{nf}$	$(\mu)_{nf}$	$(k)_{nf}$	$(\Pr)_{nf}$
0	997.1	4179	0.001002	0.58211	7.193417
0.01	1029.629	4144.072	0.001027	0.596292	7.140817
0.02	1062.158	4109.144	0.001054	0.610699	7.091315
0.03	1094.687	4074.216	0.001081	0.625335	7.044814
0.04	1127.216	4039.288	0.00111	0.640207	7.001229
0.05	1159.745	4004.36	0.001139	0.655319	6.960478
0.06	1192.274	3969.432	0.00117	0.670678	6.922491
0.07	1224.803	3934.504	0.001201	0.68629	6.8872
0.08	1257.332	3899.576	0.001234	0.702161	6.854549
0.09	1289.861	3864.648	0.001268	0.718298	6.824483
0.1	1322.39	3829.72	0.001304	0.734707	6.796956

RESULTS AND DISCUSSIONS

In the present paper a systematic study of parameters which influence the flow of CuO-water and TiO2-water nanofluid pass over a flat plate placed in magnetic field studied. It is seen that the behaviour of the fluid flow changes with the change of nanoparticle type, the results are shown in Figures 1-7.

Figure 1 Exhibits the effect of volume fraction on Prandtl Number for copper oxide - water and titania- water nanofluids. It is obvious that as Pr decreases volume fraction increases in both the cases and the decrease of Pr is more for CuO- water nanofluid than TiO_2- water nanofluid.

Figure 2: The effect of volume fraction on density for both the nanofluids are studied and it is observed that when volume fraction increases, density of all nanofluids increases and the increase is more for

Copper oxide –water nanofluid than other nanofluid.

Figure 3: Displays the effect of volume fraction on heat capacity of nanofluids .It is found that there is decrease in heat capacity as the volume fraction increases.

Figure 4: The effect of volume fraction on thermal conductivity for CuO- water nanofluid and TiO_2- water nanofluid is depicted in this figure. In both the cases thermal conductivity K_{nf} increases with volume fraction.

Figure 5: Displays the effect of volume fraction on the dimensionless temperature profile of copper water nanofluid. Here, it is noted that as volume fraction increases magnitude of dimensionless temperature also increases i,e the volume fraction is directly proportional to the dimensionless temperature

Figure 6: Shows the influence of magnetic field parameter M on the velocity profile f‘(η).As the value of magnetic parameter M increases, the retarding force increases and consequently the velocity decreases.

Figure 7: Exhibits the effect of Shear stress distribution for various value of M,It is observed that the magnitude of the wall of shear stress given by $\mathbf{(1/(1-\Phi)^{2.5})f''(0)}$ decreases when the value of magnetic parameter M increases.

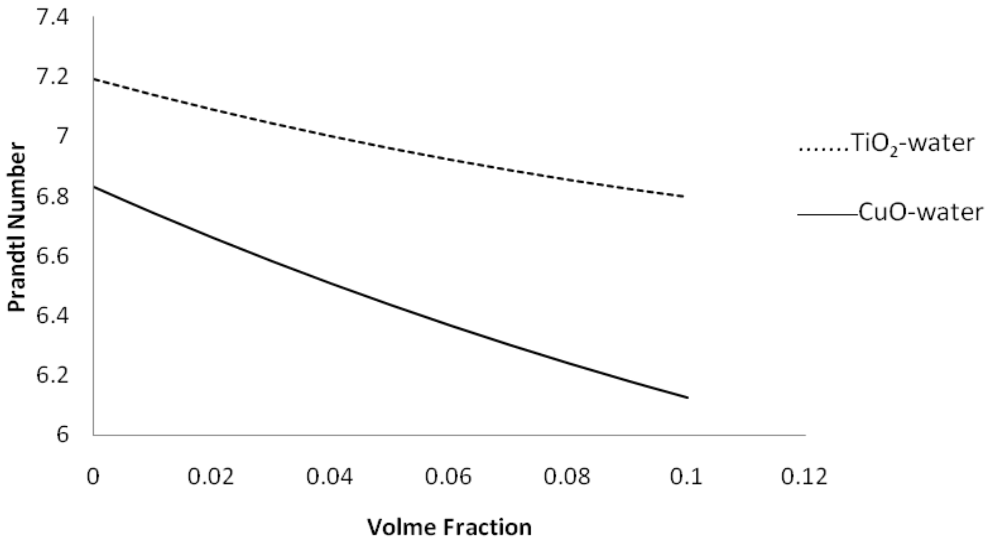

Figure 1: Effect of Volume Fraction on for Cuo Water and Tio$_2$ Water nanofluids

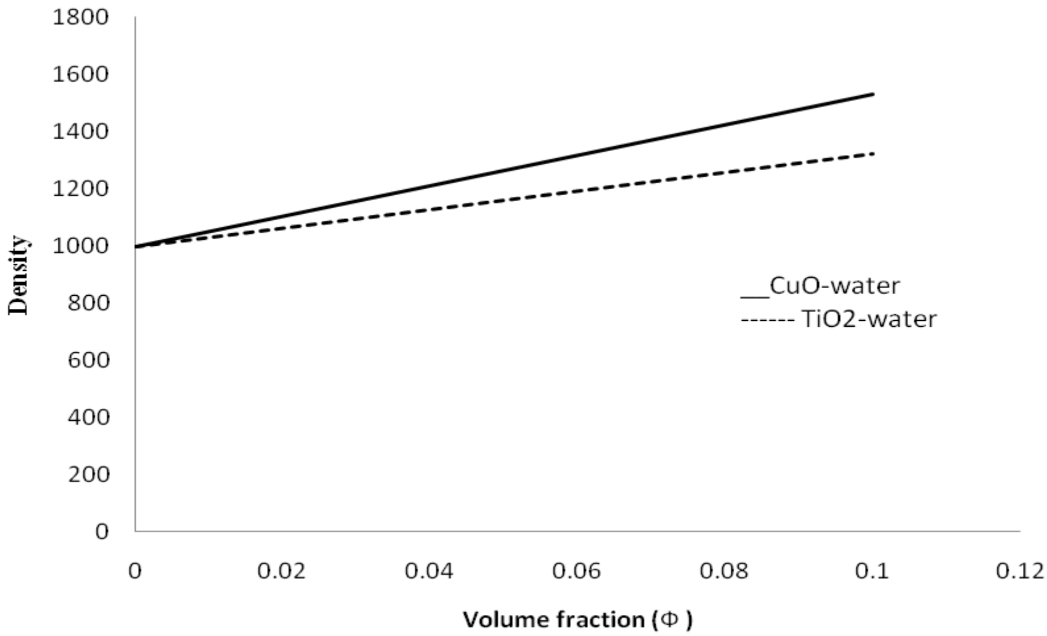

Figure 2: Effect of Volume Fraction on Density of the Different nanofluids

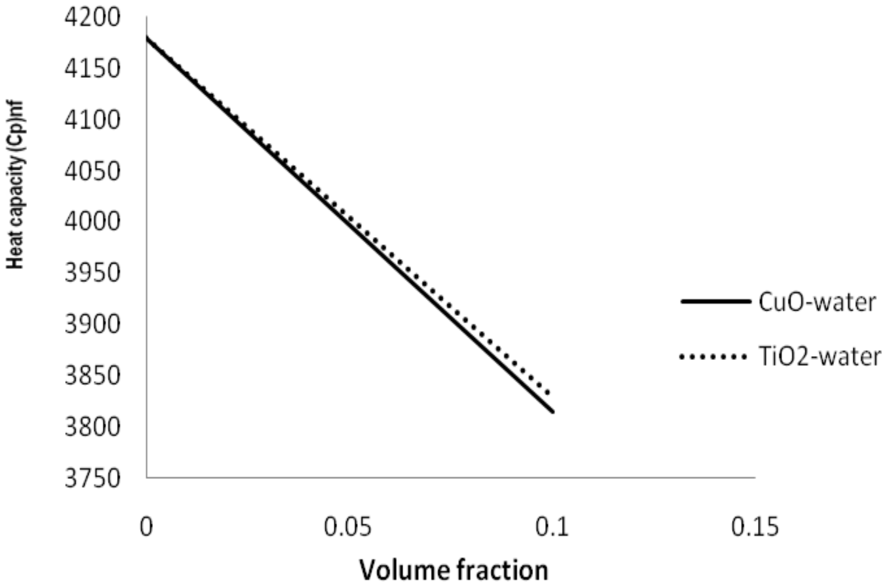

Figure 3: Effect of Volume Fraction on Heat Capacity of nanofluids

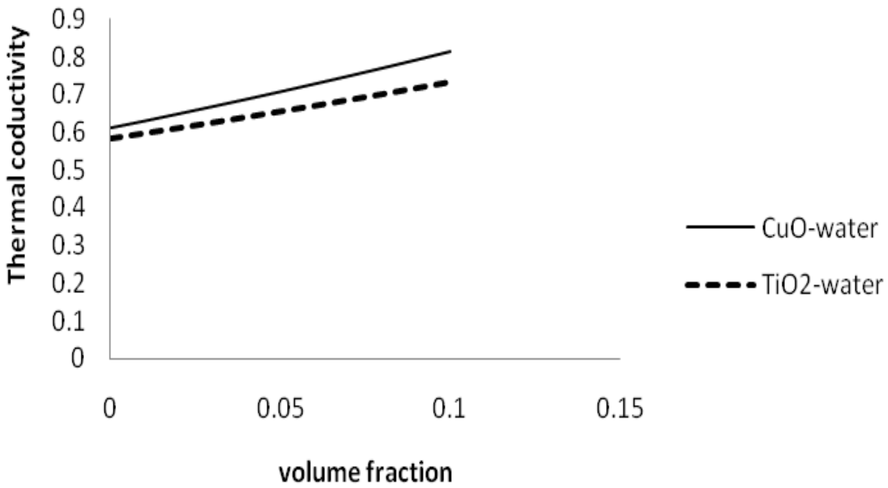

Figure 4: Effect of Φ on Thermal Conductivity (k_{nf}) of nanofluids

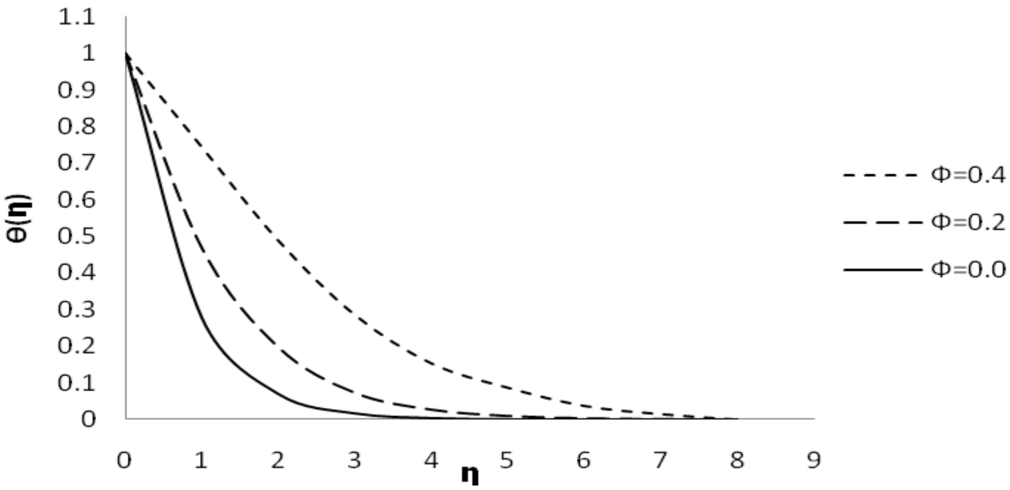

Figure 5: Temperature Profile for Various Values of Φ

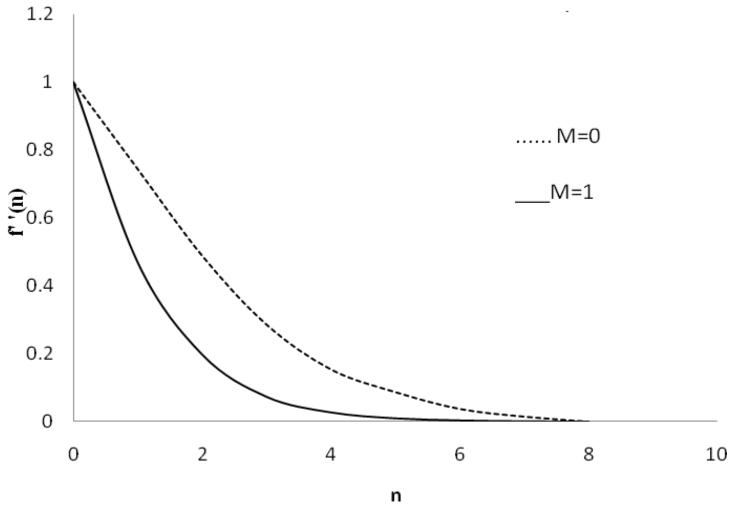

Figure 6: Effects of M on the Velocity profile F`(n)

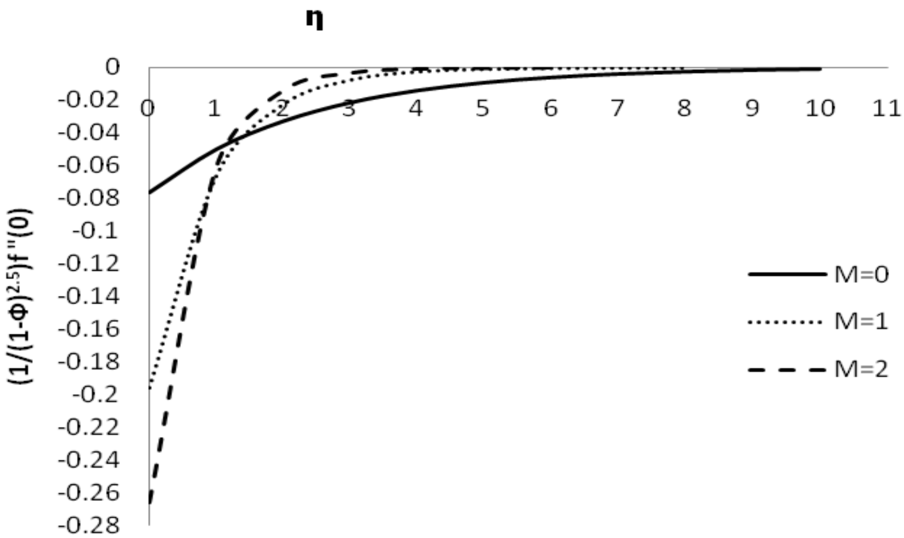

Figure 7: Shear strees distribution for various value of M

CONCLUSIONS

In this paper, the boundary layer flow of different nanofluids over a flat plate placed in a magnetic field is investigated. The governing non linear partial differential equations are transformed to ordinary differential equations using similarity transformation and solved numerically using Runge-Kutta method with shooting technique It has been observed that any increase in volume fraction of nanoparticle causes increase in density, thermal conductivity and dimensionless temperature. Whereas dimensionless velocity decreases as the value of magnetic parameter M increases. It also found that the magnitude of the skin friction coefficient decreases when the value of magnetic parameter M increases.

REFRENCES

1. **Belkin A.A. Rudyak V.Y. and Tomilina E.A. (2010).** On the thermal conductivity of nanofluids. Tech. Phys. Lett., 36(7)2660 -26662.
2. **Ahmad, S.and Pop, I.(2010);** Mixed convection boundary layer flow from a vertical flat plate embedded in a porous medium filled with nanofluids, Int. Comm. Heat Mass Transfer 37: pp 987-991.

3. **Anwar,M.I, Khan,I.S. Sharidan, S. and Salleh,M. Z.(2012**); Conjugate effects of heat and mass transfer of nanofluids over a nonlinear stretching sheet, Int. J.of Physical Sciences Vol. 7(26), pp. 4081 – 4092.
4. **Aziz, A., Uddin, M.J., Hamad,MAA and Ismail, A.I.M. (2012);** MHD flow over an inclined radiating plate with the temperature-dependent thermal conductivity, variable reactive index, and heat generation. Heat Transfer–Asian Res 41(3): pp 241–259.
5. **Choi.S(1995)** Enhancing thermal conductivity of fluids with nanoparticle.In development and application of non-Newtonian flows edited by :SiginerDA,Wang HP,66: pp 99-105.
6. **Devi, S.P.A.,Thiyagarajan, M. (2006)**; Steady non-linear hydromagnetic flow and heat transfer over a stretching surface with variable temperature, Heat Mass Transfer 42:pp 671-677.
7. **Devi,S.P.A., Andrew.J (2011);** Laminar boundary layer flow of nanofluid over a flat plate,Int.J.of Applied .Math and Mech.7(6):52-71.
8. **Hamad, M.A.A.; Pop, I and Ismail, A.I.Md.(2011);** Magnetic field effects on free convection flow of a nanofluid past a vertical semi-infinite flat plate, Nonlinear Anal.: Real World Applications 12: pp 1338-1346.
9. **Hassani, M, Tabar, M.M, Nemati, H, Domairry G and Noori F (2011)**; An analytical solution for boundary layer flow of a nanofluid past a stretching sheet. Int J of ThermScis 50: pp 2256–2263.
10. **Kandasamy R, Loganathan P and Arasu PP (2011)**; Scaling group transformation for MHD boundary-layer flow of a nanofluid past a vertical stretching surface in the presence of suction/injection. Nucl. Eng. Des. 241:pp 2053-2059.
11. **Khan WA and Pop I (2010)** Boundary-layer flow of a nanofluid past a stretching sheet. Int. J. Heat Mass Trans. 53:pp 24-29.
12. **Khare.R and Srivastava.S (2014);** MHD flow of a dusty Rivlin – Ericken Fluid through porous medium due to an impulsively Started Plate. International Journal of Advance Research and Innovation,vol 3, 2347 – 3258.
13. **Khare.R and Srivastava.S (2014) ;**MHD flow of a visco elastic dusty fluid through a porous medium induced by the motion of a semi infinite Plate. International Journal of Transactions and Applied Sciences, 6 (1), 115 – 123.
14. **Khare.R and Srivastava.S (2014);** Effect of Hall Current on MHD flow of a dusty visco elastic liquid through porous medium past an Infinite Plane. Research Journal of Mathematical and Statistical Sciences, vol 2 (10) 8-13.
15. **Matin, M.H, Heirani, M.R, Nobari and Jahangiri, P. (2012)**; Entropy analysis in mixed convection MHD flow of nanofluid over a non-linear stretching sheet. J. Therm. Sci. Technol. 7,104-119.
16. **Nadeem,S. and Lee,C (2012)**; Boundary layer flow of nanofluid over an exponentially stretching surface, Nanoscale Research Letters, **7**:94, 1-6.
17. **Nourazar S.S, Matin MH, Simiari M (2011)**; The HPM applied to MHD nanofluid flow over a horizontal stretching plate. J. Appl. Math. doi:10.1155/2011/876437. pp 181-192.